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Mathematical Model of Textronics Fabric with Textile Heater

Abstract
This work presents a mathematical description and model of textronic clothing with textile heater. The aim of this paper is to show numerical calculations of and experimental research into the compensation for thermal field disturbances that exists in the layer between the skin and clothing. A model was also created using a Matlab Simulink program. Measurements and calculation showed the qualitative agreement of the temperature drop value on the textile layers. The presented method could be used to construct an automatic temperature control system in textronic clothing.

Key words: *textronics, smart textiles, textile heater, modelling, temperature measurement, heat transfer model.*

Introduction

This work presents a mathematical description and model of textronic clothing with textile heater. This kind of textile composite can be used in special outfits or casual clothing. Its extends the clothing's traditional function of thermal compensation in the air interlayer in changing environmental conditions [4 - 6].

The aim of this paper is to show the possibility of experimental research and numerical calculations of compensation thermal field disturbances in the layer between the skin and clothing. Calculations were worked out with the use of a Matlab Simulink program [1, 2]. The research attempted to construct an automatic temperature control system for textronic clothing [3]. The thermal model, presented in Figure 1 was accepted after consideration. Figure 1 is not a cross-section of a real clothing packet but only a simple model of the three kinds of layers: two passive (air gap and textile) and one active (textile heater).

In Figure 1, $t_b = t_1$ is the body temperature, t_2, t_3, t_4 – are the boundary temperatures, $t_a = t_5$ – is the ambient temperature, R_a, R_h, R_{cl} – thermal resistances, where a – clothing interlayer, h – textile heater, cl – textiles, R_{r+c} – thermal resistance of heat emission during convection and radiation, q^l, q_e – heat flux density, d – layer thickness.

The cylindrical structures the physical model (Figure 2) were the subject of our considerations. The cylindrical heater simulated the human body and the next layers were: air gap, textile heater and clothing layers closely packed [8]. In this work the authors considered only heat transfer in steady conditions without the humidity transfer mechanism.

Theoretical considerations of the heat transfer mechanism

Three cases of constant thermal state were considered [9]. In the first case, the textile heater was switched off and the ambient temperature was steady. In the second case, we assumed that the surrounding temperature dropped when the textile heater was still switched off. This caused a change in temperature distribution in the layers considered. In the third case, the textile heater was switched on, in order to compensate the thermal disturbance.

In the first case, the density of the thermal flux transmitted through the layers of clothing from the human body to the surroundings may be expressed by the following equation:

$$q_\lambda = \frac{t_1 - t_4}{R_a + R_h + R_{cl}}, \text{ in W/m}^2 \quad (1)$$

The heat resistance R_h and R_{cl} were calculated from the model parameters. The thermal resistance of the air interlayer R_a was defined from similar criteria [11].

In the conditions considered the Grashoff number is equal Gr = 0.264 and Prandtl number is equal Pr = 0.701, thus GrPr = 0.185 and GrPr < 10^3. For these conditions the thermal resistance of the air gap could be calculated considering only the thermal conduction phenomenon.

The thermal flux density transmitted to the surroundings may be expressed by the following equations (2), (3):

$$q_e = \frac{t_4 - t_a}{R_{r+c}}, \text{ in W/m}^2 \quad (2)$$

and

$$R_{r+c} = \frac{1}{\alpha_c + \alpha_r}, \text{ in Km}^2\text{/W} \quad (3)$$

where: α_c is the convective heat transfer coefficient, α_r is the radiation heat transfer coefficient.

The coefficient of convective heat transfer α_c was calculated in accordance with the Michiejew procedure, for forced convection [11]:

$$\alpha_c = Nu \frac{\lambda_a}{l}, \text{ in W/m}^2\text{K}, \quad (4)$$

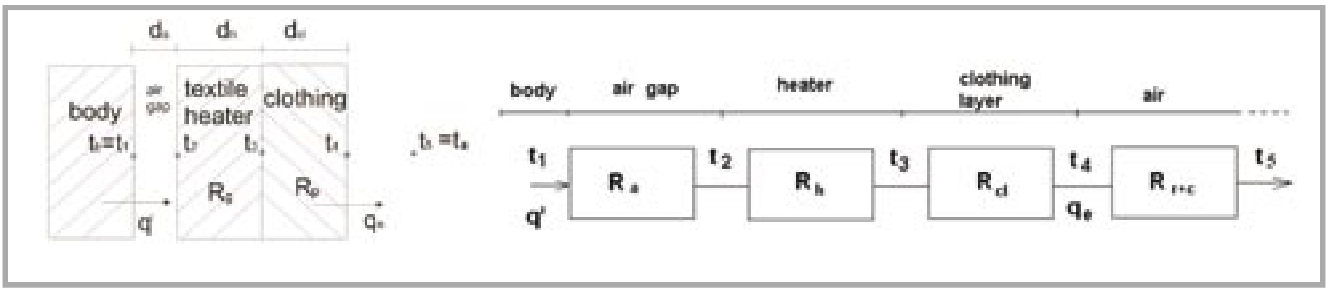

Figure 1. The model investigated: (A) Thermal model with active textile heater; (B)Thermo resistant scheme of a textronic fabric.

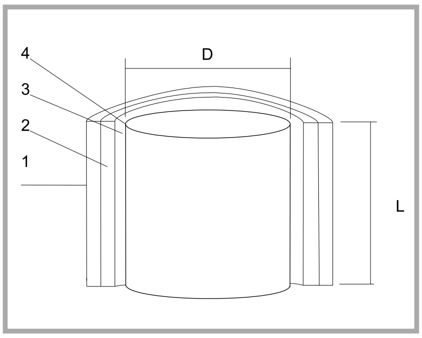

Figure 2. Geometrical research model; 1 – cylindrical heater, the source of heat as an equivalent of the human body, with surface temperature controlled at 32 ± 0.2 °C [8, 15], dimensions: $D=96\cdot10^{-3}$ m, $L=280\cdot10^{-3}$ m, 2 – air gap, 3 – textile heater, nonwoven iron fabric Bekinox, $d_h=2.1\cdot10^{-3}$ m, thermal conductivity $\lambda_h = 0.051$ W/m K, 4 – clothing layer of wool fabrics, thickness $d_{cl} = 1.57\cdot10^{-3}$ m, thermal conductivity $\lambda_{cl}=0.043$ W/m K.

where: λ_a is the air thermal conductivity, l is the linear dimension of the research object.

$$\mathrm{Nu} = C\,\mathrm{Re}^n, \qquad (5)$$

where: Nu is the Nusselt number, Re is the Reynolds number, C and n depends on Re.

The value of Nu, Re number and λ_a were calculated for the average temperature which equals $t_{av} = (t_4 + t_a)/2$, in °C.

The radiation heat transfer coefficient α_r was calculated from Boltzman's low:

$$q_r = C_0\varepsilon\left[\left(\frac{273+t_4}{100}\right)^4 - \left(\frac{273+t_a}{100}\right)^4\right], \quad \text{in W/m}^2, \qquad (5)$$

where:
- C_0 – the radiation coefficient of a technical black body,
- ε – the total emissive coefficient (in our calculation assumed as $\varepsilon = 0.9$) [7, 10, 13],
- t_4 – the textile surface temperature,
- t_a – the surrounding temperature.

The coefficient α_r was described by equation (6):

$$\alpha_r = \frac{q_r}{t_4 - t_a}, \quad \text{in W/m}^2\text{ K} \qquad (6)$$

Temperature distributions between all layers were calculated at a steady ambient temperature of $t_a = 26.2$ °C, for thermal equilibrium $q_e = q_\lambda$. The calculation was conducted in the same way for the second case, where the ambient temperature was decreased to $t_a = 18.7$ °C.

In this case the temperature felt by the body was lower, because the temperature t_2 on the textile heater's surface dropped by 1.7 °C, and thermal comfort got worse.

In the third case, the thermal equilibrium parameters were calculated with the inclusion of an active textile heater. After switching it on, the boundary temperature t_2 of the textile heater arrived at the previous state. The authors assumed that the thermal equilibrium between the skin and the textile heater was the same as the one previously established, and such was the thermal comfort feeling.

In describing this case, the thermal flux density emitted to the surrounding was equal to the sum of flux density q^I_λ emitted from the body, and the flux density emitted from the textile radiator q_h. The thermal balance for a textronic set of textile layers with an active heater could be described using the formula:

$$q^I_\lambda + q_h = q_e, \quad \text{in W/m}^2, \qquad (7)$$

where $q^I_\lambda = q_\lambda$ is the thermal flux density, in the first case. The geometrical model taken into consideration, is shown in Figure 2.

Within the scope of preliminary investigations, the basic properties of the clothing packet were determined, and the coefficients of thermal conductivity and thermal resistance were assessed.

A constant air gap was obtained by installing a distance net made from technical polyurethane of thickness $d_a = 2\cdot10^{-3}$ m and thermal conductivity

$$\lambda_{a(eqv)} = 0.030 \text{ W/mK};$$

the surface of polyurethane used to built the distance net was equal to $142\cdot10^{-6}$ m^2. The distance net was also electrical insulation between the textile heater and the human body (metallic cylinder). The equivalent thermal resistance of the real air gap, taking into account the fact that the polyurethane net is equal to 0.0442 m^{2}K/W. The ratio of external diameters D_{out} to internal diameters D_{int} of the model (Figure 2) was smaller than 2. Thus, the calculations could be the same as for a flat sample [12].

The particular drop of temperature on the passive layers when the textiles heater was switched off, was calculated as

$$\Delta t_{i,i+1} = q \cdot R_{i,i+1}, \quad \text{in °C} \qquad (8)$$

where $i = 1, 2, ..., n$.

The heat flux density generated in the textile heater grew linearly from 0 to q_h, in the direction of the external layer.

During proper functioning of the textile heater, the temperature t_2 could not be larger than the temperature of the skin t_1. The heat flux is always in an outwardly direction.

The authors received the value q_h from a solution of the two equations (1) and (2) for two surrounding temperatures without heating, as the difference of the steady heat flux density, which is presented in Figure 3.

The equation (9) of the heat flux density $q(x)$ in the textile heater's layer has the form:

$$q(x) = q^I_\lambda + \frac{q_h}{d_h}x, \quad \text{in W/m}^2 \qquad (9)$$

For the boundary condition $t(x)_{x=0} = t_2$, using Newton's equation after integrating the value of the temperature drop, which we obtained for the textile heater, it may be described by equation [14]:

$$\Delta t_{23} = t_2 - t_3 = q^I_\lambda\frac{d_h}{\lambda_h} + \frac{1}{2}q_h\frac{d_h}{\lambda_h}, \quad \text{in °C} \qquad (10)$$

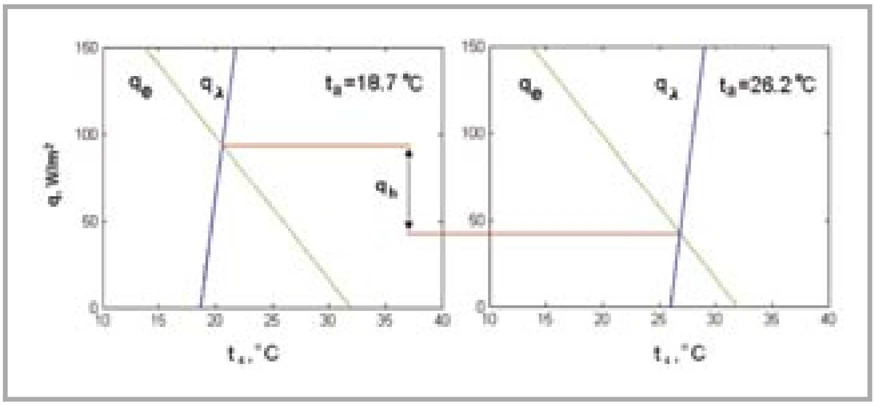

Figure 3. Graphical interpretation of calculated results received from a Matlab Simulink program.

The measuring stand

The thickness of the textile packet elements were measured using a Tilmet 73, an optoelectronic thickness gauge with accuracy class of 0.5%. Thermal conductivity measurements were carried out using a Tilmet 75 conductometer of the two-plate-Poensgen type, with inaccuracy of 0.001 W/mK. The heat power P_h was measured with an inaccuracy of 0.02 W. The boundary temperature was measured with a type J thermocouple with wire diameter of 0.2 mm, which co-operated with use of an AD 594AQ integrated circuit. Inaccuracy of temperature measurement was up to 0.2 °C. Temperature distribution on the sample surface was measured using a DM53 radiation pyrometer equipped with a sensor from Cole Parmer Infrared Co, with reading inaccuracy of 0.1 °C. The ambient temperature was measured using a liquid laboratory thermometer with an elementary elementary scale of 0.2 °C. The relative moisture of the air was measured with an inaccuracy of 1% RH. The air speed was measured using a mechanical anemometer with an inaccuracy of 0.2 m/s. This last measurement was carried out in order to confirm the stability of the measurement conditions.

In this work the authors only analysed temperature differences. A precise, absolute value of temperature was not necessary in the considerations presented. The indications of all the measuring instruments were unified in one of the thermometers. A liquid thermometer was chosen for its small random error. The thermometer was calibrated with the assistance of MicroCal 100+ calibration, Eurotron company, with an accuracy of reading ±0,15 °C. Suitable corrections from bias were calculated.

Verification of the mathematical model

To verify the mathematical simulation described, shown in Figure 2, a physical model was built with parameters used in calculations. A metal cylinder was used as a source of heat (equivalent of the body) surrounded with a textile packet. The metal cylinder was built in accordance with Polish standard PN-86/P-04617, "Textile heat insulation. Determination of energy flux density"[16]. The cylinder was warmed up with a thermostatic liquid controlled with an inaccuracy of 0.2 °C. The results of calculations and measurements of the average values are presented in Table 1. They show the temperature drop for particular boundary textile layers for two surrounding temperatures of 26.2 °C without heating and of 18.7 °C with and without heating. The worsening thermal comfort of the textronic clothing was affected by a surrounding temperature drop of over 7.5 °C, and a drop in the interlayer temperature t_2. This phenomenon we tried to eliminate by switching on the textile heater. The experimental and mathematical results obtained are also presented in graphic form, in Figure 4.

The temperature drop Δt_{34} on the outer layer was calculated from measurements using a radiation pyrometer. The emissive coefficient of the textile fabric assumed arbitrary acceptations $\varepsilon = 0.9$ [7, 10, 13].

The surface of the woolen fabric was porous and had a rough texture which enlarged the inaccuracy of the measurement [13]. Thus, to analyse the agreements of the results obtained from measurements and calculations, only the temperature drops Δt_{12} and Δt_{23} were compared.

Table 1. *The temperature drop for particular boundary layers of textronic system.* ***Constant:*** *RH = 65%, air speed V = 2 m/s, t_1=32 ± 0.2 °C.*

Cases	Ambient temperature, t_a	Power of heater P_h	Calculated value			Investigated results		
			Δt_{12}	Δt_{23}	Δt_{34}	Δt_{12}	Δt_{23}	Δt_{34}
	°C	W	°C	°C	°C	°C	°C	°C
1	26.2	0.00	1.6	1.2	1.7	1.6	1.1	1.7
2	18.7	0.00	3.3	2.5	3.3	3.7	1.5	4.6
3	18.7	2.54	1.7	2.3	2.8	1.7	1.5	4.0

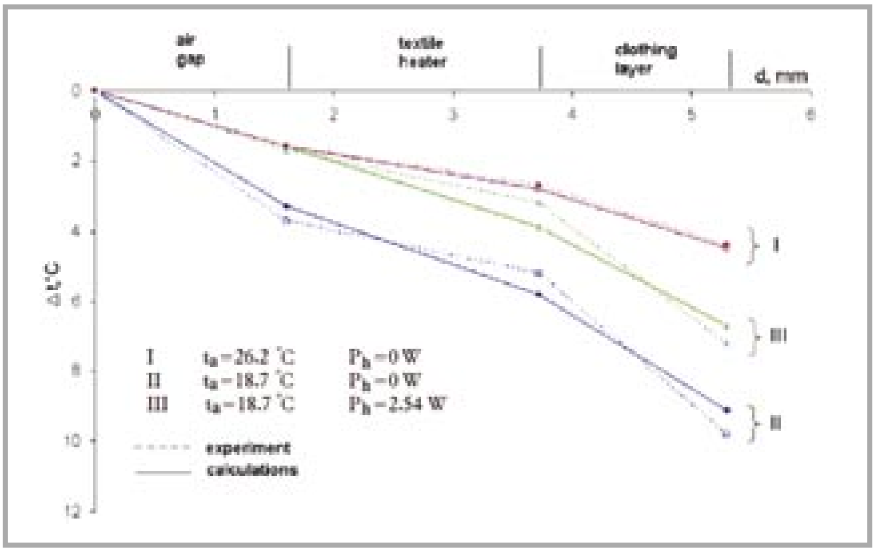

Figure 4. *The temperature drop on layers considered, for three cases; I) First case for surrounding temperature 26.2 °C without heating. II) Second case for surrounding temperature 18.7 °C without heating. III) Third case for surrounding temperature 18.7 °C with heating.*

Conclusions

- The drop in ambient temperature was caused interlayer temperature disturbances. The possibility exists to compensate them by switching on the textile heater. This restores the previous underclothing temperature, which is represented by value Δt_{12} in Table 1 and Figure 4.
- The model described in the Matlab Simulink program which enables quick definition of the influence of the object parameters and calculation of the heat parameters.
- Measurements and calculations showed the qualitative agreement of the temperature drop value on the textile layers. This shows the correctness of the thermal model.
- The differences in calculation and measurement result from the differences considered, on average about 0.3 °C, which is a relatively small

value (Table 1, Figure 4). The model can be used to design an automatic control system with a textile heater as an actuator for underclothing temperature control.

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